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A ROBUSTNESS ANALYSIS TOOL FOR FLEETWIDE VARIABILITY AND DEGRADATION ASSESSMENT IN PROPULSION CONTROL SYSTEMS

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ABSTRACT
The desire to enlarge the flight envelope of tactical aircraft and provide enhanced maneuvering capabilities has led to the use of forces and moments produced by the propulsion system to directly influence aircraft dynamics. This coupling between propulsion and flight dynamics is significant enough that traditional techniques for control system design and analysis are either conservative or inaccurate. An integrated approach is required in order to obtain an overall system that provides stability and performance with minimum pilot workload. The authors propose a solution that integrates existing techniques for linear robustness analysis, nonlinearity analysis, optimization, and robust identification into a software tool that facilitates the analysis of robustness of stability and dynamic performance of propulsion control systems and investigates the system's ability to meet aggregate performance measures, specifically in the presence of fleet-wide component "variability." The paper first discusses the motivation for the solution by identifying the variability problem in control system design. Following this, an overview of the proposed solution is presented with highlights of key elements, including a discussion on nonlinearity assessment techniques. The paper next describes the prototype version of the software tool and its initial analytical capabilities. Results of the prototype as applied against a nonlinear, engine control model using two different optimization routines, Genetic Algorithms and Particle Swarm Optimization, are presented to demonstrate the promising performance of both algorithms for finding the worst variability due to operating condition flight dynamics and aero-thermal component degradation.

MOTIVATION
Traditionally, verification and validation of closed loop robust stability and performance of propulsion systems is accomplished through modeling, analysis, and simulation. Central to this process is an analysis of the effects of simultaneous variations in performance and stability related parameters on the overall system stability margins and dynamic performance. The concept of variability arises because an exact system model would need infinite dimensions, exact values of system parameters, and a comprehensive external disturbance model. The limitations of the model, along with fleet-wide and time-driven variability, introduce uncertainty in the system behavior that manifests through changes in stability and performance. The following are some probable sources of variability suggested by the authors:

1. **Fleet-wide**: This type of variability arises due to tolerances within each component and the cumulative effect of the variabilities of multiple components in an assembly. The effects of small, simultaneous component-level variation on the entire system can lead to large variation in performance from one engine to the other.
2. **Usage-based Degradation**: Component degradation has an equally important influence on variability. Fault tolerant control requirements specify condemning criteria for components individually. However, it is possible that the combined net effect of simultaneous performance variation in multiple components may lead to sub-par performance before individual components reach their respective condemning criterion.
3. **Flight Envelope Nonlinearity**: Operating conditions of nonlinear systems also lead to variability in component and system performance. This type of variation can be estimated

by defining the performance boundaries of the system and calculating variability at set points that represent extreme or worst-case behavior.

In addition, there are also traditional sources of variability, such as sensor noise and external disturbances, to be considered. Variability introduced through the various sources mentioned, in any combination, brings about uncertainty and unpredictability in the behavior of a system, causing it to perform below the desired level. The authors present three categories of probable direct effects of variability:

1. **Stability:** Stability, loosely defined as the ability of a system to provide a bounded response for a bounded control input, is the most critical criterion affected by variability. For a closed loop system, variability influences degraded health and sluggish response and may cause the controller to overcompensate, resulting in instability. In contrast, in open loop system control, stability is often improved but at the cost of performance. Stability margin metrics, which include gain margin, phase margin, and MIMO Singular Value margins, provide a measure for how much variability a system can tolerate before becoming unstable.
2. **Time Domain Performance:** Variability introduced through component aging leads to degraded time domain performance, affecting performance metrics such as rise time, setting time, and percent overshoot. These metrics are commonly considered classical step response characteristics.
3. **Frequency Domain Performance:** Frequency domain performance also becomes degraded and is reflected through metrics like bandwidth and noise rejection capability.

The challenge lies in characterizing the effect of simultaneous variation on control system performance and stability. The variability problem becomes increasingly complex in an integrated flight and propulsion system due to the wide range of power providing and flight control components, each of which can vary in different ways. The problem is further compounded by the various sources of variability for each individual component. To address these complexities, robustness analysis must explore a multi-dimensional parametric “space” of all variations to quantify changes in stability and performance metrics and identify “regions” within the space of sub-optimal system performance.

Current variability analysis techniques act directly upon a system model. The variability of the system is captured through variations in individual component parameters and through aggregate parameters that model noise and external disturbances. The essence of robustness analysis lies in exploring the parametric space of individual component-related parameters and aggregate parameters to find the worst-case combination for stability and performance for expected operational modes. For Single Input Single Output (SISO) linear systems, exploring parametric space to determine robustness may be reduced to techniques like root locus and Nyquist criteria. For large order, MIMO-type systems, singular value techniques like μ analysis and generalized singular value analysis are common. Most large order MIMO analysis techniques are formulated with a linear model of the system and solve a nonlinear optimization problem. However, the current techniques face the following drawbacks:

1. **Linearization of Nonlinear Systems:** There are no known analytical techniques that are applicable to large order, nonlinear systems. Linearization provides a good measure of behavior at the point of linearization, but poor estimates at operational points away from linearization.
2. **Uniform Distribution of Variable Parameters:** The key to may of the aforementioned analysis techniques is the assumption of a uniform distribution of variable parameters. In reality, the distribution may be more suitably Gaussian.
3. **Convex Mapping between Parameter Space and Stability/Performance Metric Space:** This is an often ignored drawback and that is not discussed in literature. All variability modeling assumes a variability envelope. For example the nominal efficiency of a compressor may vary from 0% to -5%. Variability modeling for these analytical techniques assumes that the change in behavior between 0% and -5% efficiency forms a convex set. This may not be true for nonlinear systems and can lead to erroneous conclusions about stability and performance of intermediate points, especially in multi-dimensional parametric space (i.e., during simultaneous variation of parameters).
4. **Convex Mapping between Linearized Operating Point Space and Stability/Performance Metric space:** Similar to the assumed convexity of the multi-dimensional parameter space to stability/performance space mapping, the analytical techniques assume convexity of mapping in the linearization process, i.e., the response of the system in between any two linearization points is either a linear combination of the two behaviors or is within a convex region beneath the linear combination. An interpretation of the linear combination assumption implies that if the system is linearized at two operating points, then the poles of every operating point in between lie on a straight line joining the respective poles from the two linearized operating points. This is not a valid assumption for any nonlinear system.
5. **Computationally Intensive:** In addition, statistical sampling driven techniques, like Monte Carlo simulation, are commonly used to estimate the effect of simultaneous perturbations in a MIMO system. However, these techniques suffer from the so-called ‘curse of dimensionality,’ where an increase in the number of perturbations exponentially increases the number of computations required.

SOLUTION

The proposed solution draws upon the existing techniques of linear and nonlinear analysis, robust identification, and optimization. The overall approach is shown in Figure 1.

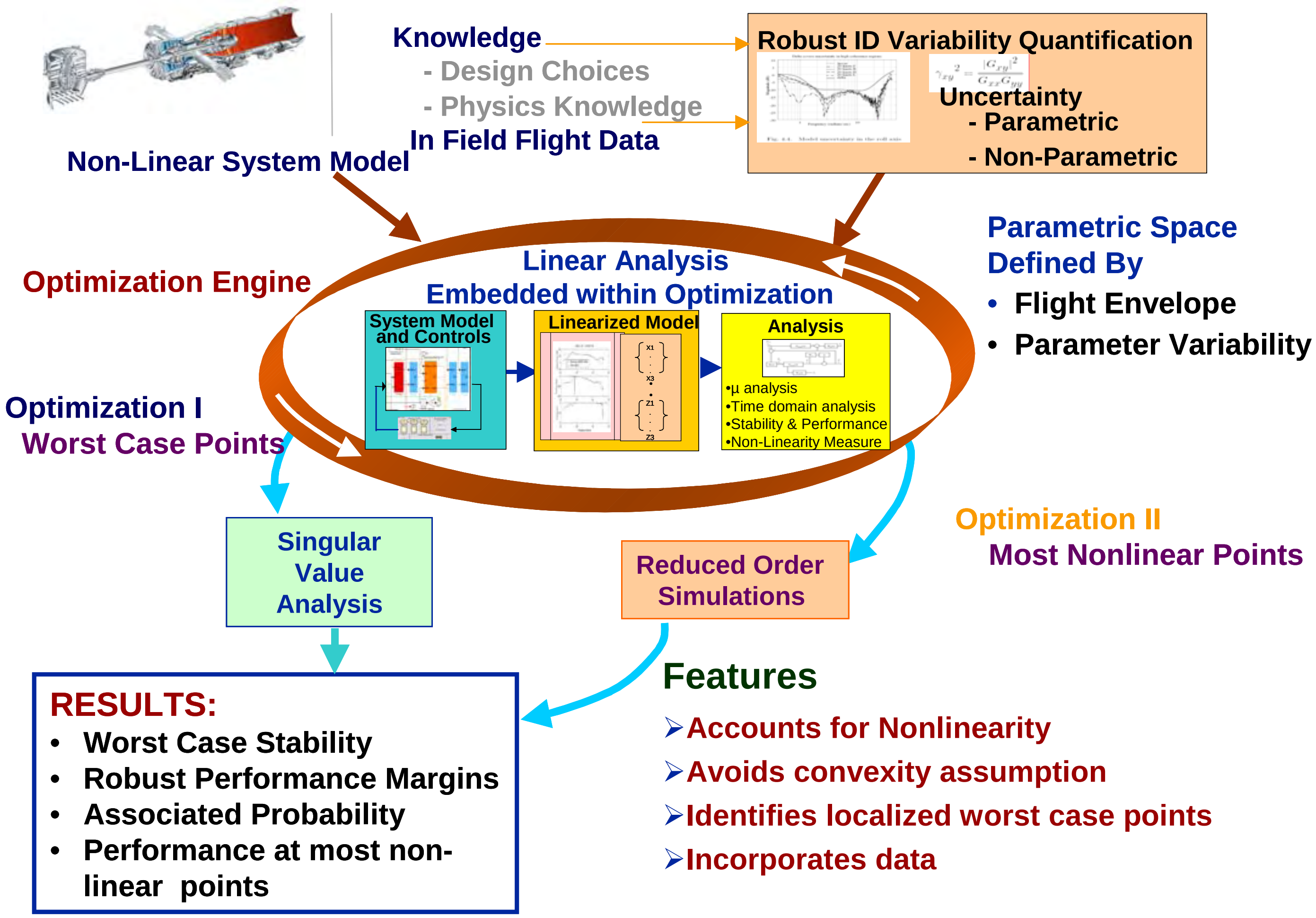

FIGURE 1: SOLUTION: ROBUSTNESS ANALYSIS TOOL

This approach combines applicable techniques and evolves them to solve the problem. This is in contrast to current research work, where the focus is on evaluating each technique [2]. Due to the large order, nonlinear nature of the system and the absence of nonlinear robustness analytical techniques, linear analysis is still an integral part of the approach. Traditionally, application of linear analysis to a nonlinear system usually involves developing a grid of operational points and using linearized versions of the nonlinear model to solve the robustness analysis problem\ close to the linearized point. This leads to the creation of linear subspaces, within which the linear analysis is valid. The previously stated drawbacks one, three, and four are inherent in grid-based linearization; therefore, the proposed solution adopts optimization to drive a search process to simultaneously identify the most critical/worst case linear subspaces in the operation envelope and perturbed parametric space in an intelligent order. These critical/worst cases linear subspaces may be defined by:

- Poor linear robust stability margin or unstable behavior
- Poor linear robust performance margin or sub par performance
- Highly nonlinear behavior that cannot be characterized accurately by linear stability/performance metrics

Optimization is central to the solution. The optimization search is divided into two levels. Optimization I involves simultaneously exploring the flight operation and parameter perturbation space to identify the worst case operating point(s) for stability and performance using linear robustness analysis. Subsequent to this first stage search, singular value analysis is performed around the identified worst case operating point(s) in order to obtain greater understanding of the time and frequency domain characteristics of these subspaces. Optimization II seeks to identify highly nonlinear subspace regions within the flight envelope and component parameter variability space. These regions can then be evaluated for robustness through reduced order simulations. Again, the need to isolate and observe points that are highly nonlinear stems from the accuracy limitations associated with modeling nonlinear behavior through linearization. An important feature of the proposed solution is the incorporation of actual test cell or flight data into the analysis process to enable realistic quantification of system variability. Variability quantification is the process of taking the nominal value of component parameters and assessing their variance or range across the life of an entire fleet. This process is called parametric quantification. Parametric quantification based analysis for a

MIMO system is computationally intensive. An alternate step in this process is to aggregate the variability of all these parameters and convert it to nonparametric variability. This aggregation in multi-dimensional parametric space leads to more conservative [3], but less computationally intensive and conclusive analysis. Nonparametric variability or uncertainty modeling techniques are recommended for use to make up for un-modeled system dynamics and the effects of sensor and actuator noise. Nonparametric uncertainty can also accurately represent a nonlinear system for linear analysis, thus reducing the complexity.

The proposed solution, shown in Figure 1, presents the following advantages:

1. **Avoids Convexity Assumption:** Addresses the previously mentioned drawbacks three and four by incorporating optimization to search for worst case points in various subregions.
2. **Identifies Worst Case Points in Localized Regions:** The optimization based search identifies worst case performance points, thus providing critical regions for further analysis without the cumbersome grid search or computationally intensive Monte Carlo simulations.
3. **Incorporates Variability, Nonlinearity and Uncertainty Modeling:** Incorporates Robust Identification as a method to model and quantify uncertainty due to parametric variation and modeling assumptions.
4. **Equipped to use Multiple Metrics:** The optimization search is highly configurable to find worst cases for various time and frequency domain control metrics such as stability margin, performance margin, rise time. Or the user may define a custom metric.
5. **Uses COTS Tools:** Most of the techniques mentioned can be developed using Commercial Off The Shelf packages like MATLAB and MatrixX, thus avoiding the complexity of using programming languages for development and testing.
6. **Analyzes for Nonlinearity Severity:** Addresses previously mentioned drawback one by integrating the optimization search process with techniques for assessing nonlinearity (as described in the following section).

The proposed solution also presents some challenges. The main risk associated with this process is the assumption that optimization will find the worst case and most nonlinear points in each sub region. This assumption is associated with most optimization problems and change in tolerance and grid spacing can help minimize this risk. Another drawback is the inherent conservatism associated with most singular value approaches. The singular value approach adopted initially is μ analysis. This is a conservative technique and may be substituted by a generalized singular value measure or even H_2 [4].

Measuring and Identifying Nonlinearity for Analysis

The need to measure and analyze points that are highly nonlinear arises because analysis of the linearized model may not give an acceptably accurate description of nonlinear behavior. Part of the strategy in this solution is to implement an optimization routine algorithm that searches both the flight operational envelope space and the parametric space to identify the most nonlinear points. The optimization is capable of partitioning the search space and finding a series of local minima, thus identifying the most nonlinear points in each region and performing a one-off simulation at each of these points to verify stability and performance. This process is especially important on the peripheries of the fleetwide variability envelope where efficiencies, frictions, and stictions can create discontinuities in the response, thus representing a large nonlinearity.

Consider a general nonlinear multivariable system with input u, output y, and states x, where input u belongs to the space of admissible input signals Ua. The system and its input/output behavior are given by:

$$\begin{aligned} \dot{x}(t) &= f\left(x(t),u(t)\right) \\ y(t) &= g\left(x(t)\right) \end{aligned} \quad \text{(Eq.1)}$$

or simply $y = N[u]$

Definition 1 [5, 6]: The nonlinearity measure ϕ of a stable I/O system N, for input signals $u \in U_a$, is defined by the non-negative number:

$$\phi = \inf_G \sup_u \frac{\left\| G[u] - N[u] \right\|_p}{\left\| u \right\|_p} \quad \text{(Eq. 2)}$$

with G being a linear operator that represents the same input/output system and $\| \bullet \|_p$ as a norm in the space of the argument. The size of the nonlinearity of a nonlinear system is thus defined as the normalized largest difference between the outputs of the nonlinear system, *N*, and a linear system, *G*, as measured by the norm, $\| \bullet \|_p$. The difference is taken with respect to the worst case input in U_a. An example for a specific and useful choice of nonlinearity measure is obtained by restricting U_a to consist of L2 signals with bounded energy $\| u \|_{L2} < \varepsilon$ and the norms being the L2-norm.

Computation: In general, the computation of nonlinearity measure requires the solution of a nonlinear, infinite dimensional min/max problem of high complexity. However, it is possible to compute approximate solutions with reasonable accuracy. Without losing generality, consider a SISO system with input *u* and output *y* satisfying:

$$\begin{aligned} &\lim_{T\to\infty} \|u\|_2 < \infty; \lim_{T\to\infty} \|y\|_2 < \infty \\ &\|y\|_2 = \sqrt{\frac{1}{T}\int_0^T y^2(t)dt} \end{aligned} \quad \text{(Eq. 3)}$$

Consider the special class of input signals:

$$u(t) = A\sin(\omega \cdot t) \in Us. \quad \text{(Eq. 4)}$$

Any nonlinearity measure based on U_s instead of U_a will have a lower bound on the actual measure, but will have reasonable accuracy. In particular, the following approximate nonlinearity measure χ for *N*,

$$\chi = \sup_u \inf_G \lim_{T\to\infty} \frac{\|G[u]-N[u]\|_2}{\|u\|_2} \quad \text{(Eq. 5)}$$

χ is a lower bound on ϕ, or $\chi \le \phi$, within a confidence level of 90%. This can be computed by:

$$\chi = \sup_{A,\omega} \frac{1}{A}\sqrt{A_0^2(A,\omega) + \sum_{k=2}^{\infty} \frac{A_k^2(A,\omega)}{2}} \quad \text{(Eq. 6)}$$

where $A_k(A,\omega)$ are the amplitude coefficients of the k^{th} harmonics of the steady-state response to input $u(t)$. Calculation of the lower bound thus only involves computation of Fourier coefficients for different periodic output signals. For simple nonlinearities, analytic calculation is possible, whereas for systems that are more complex, it is possible to use efficient numerical techniques.

A gradient-based nonlinearity measure algorithm currently under development by the authors uses pole migration in the vicinity of each point as a measure of nonlinearity at that point. This algorithm is based on the assumption that in a highly nonlinear operational region, the poles of the linearized system will exhibit far greater variation, than in a relatively linear region. This algorithm uses existing linearization and pole calculation algorithms to estimate the poles at an operating point. It then uses the optimization routine in a small subspace around that operating point to find the maximum pole migration gradient. This gradient is a measure of the system nonlinearity.

Optimization Formulation

To evaluate the robustness properties of a nonlinear system using the linear subspaces, it is necessary to determine the most critical points in the operation envelope and in the perturbed parametric space. The approach can be summarized as a two step procedure:

1. Optimize a set of stability and performance index functions in the combined parametric and operational subspaces (with respect to plant constraints) in order to determine the worst case operating points (with respect to stability and performance). This process may involve an optimization for each index function and will provide a set of points for further detailed analysis,
2. Optimize a nonlinearity measure and rank a set of most nonlinear points in the combined parametric and operational subspaces in order to evaluate through simulation the behavior of these points.

Consider the closed loop nonlinear system shown in Figure 2 with states x, input u, and output y. The linearized model that represents the nonlinear system is

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \quad \text{(Eq. 7)}$$

The complementary sensitivity function of the linear model is defined as the transfer function from the reference input to the output:

$$T(s) = C(sI - A)^{-1}B + D \quad \text{(Eq. 8)}$$

The sensitivity function of the closed loop linear model is defined as the transfer function from the reference input to the output error:

$$S(s) = 1 - T(s) \quad \text{(Eq. 9)}$$

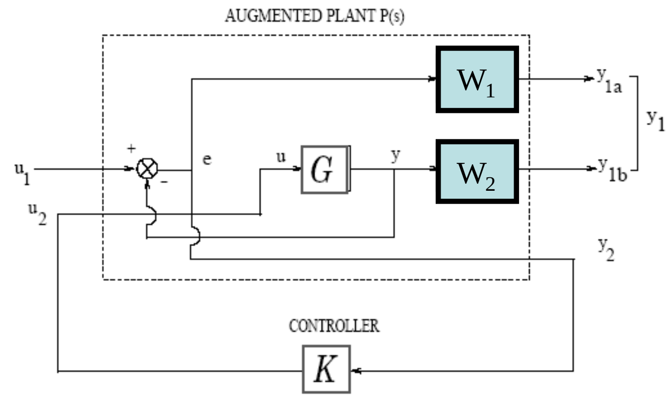

FIGURE 2: SENSITIVITY AND COMPLIMENTARY SENSITIVITY FUNCTION FOR CLOSED-LOOP SYSTEMS

In the robust control framework, two types of weighting functions are used in the frequency domain (s domain) to shape the sensitivity and complementary sensitivity functions. The performance weight, W_1, penalizes the low frequencies because time domain performance specifications are reflected in the low frequency behavior of the system, as discussed later in this section. The uncertainty weight, W_2, penalizes the high frequencies because modeling errors due to linearization create a linear model that neglects high frequency dynamics. Therefore, in general, W_1 has low-pass filter characteristics, whereas W_2 is usually a high-pass filter. In the analysis of robust stability and robust performance of a linear model, the H_∞ norm of W_1S measures whether or not the system performs robustly and the same norm of W_2T gives a measure of how stable the system is in the presence of modeling uncertainty. The estimations of robust stability and performance are defined from these measures. A value greater than one indicates that the system is unable to meet its required stability or performance measures, while a value of one implies marginal performance or stability. For values below one, the system provides robust stability or performance with robustness margin proportional to the inverse of the metric.

Optimization Approaches

Two popular approaches, Genetic Algorithms, and Particle Swarm Optimization, were used in the initial application, to demonstrate the promising performance of the solution and compare the performance of the two methods to find the worst case stability and performance due to operating condition flight dynamics and aero-thermal component degradation.

Genetic Algorithms (GA) are inspired by the notion of evolution through natural selection. The basic idea is to resolve an optimization problem by making the solution evolve from an initial set of random solutions. The genetic algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm selects individuals at random from the current population to be parents and uses them produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution [7].

The Particle Swarm Optimization algorithm (PSO) creates a number of "particles" in the n-dimensional search-space, with each particle initially having a random location. Each particle is aware of the "particle minimum," which is the location where the particle encountered the minimum value of the cost function or selected stability/performance metric. In addition, each particle also has information on the "global minimum," which is the minimum value encountered by any of the particles. In addition, a random acceleration may also be specified for each particle at every iteration. At the end of the iteration cycle (specified by the number of iterations), the location of the overall minimum value of the objective function is output as the optimal setting for the parameters. For searching the parametric and flight envelope space, particle swarm has the distinct advantage that it allocates a particle to each of the corner points of the region, thus covering the most extreme, and typically the most nonlinear, areas of each region. Apart from attempting to converge to the global minima, PSO also stores a pre-specified number of worst-case points. Both of these aspects are important in robust analysis and reduced order simulation of the worst-case points that may be close to violating the stability or performance constraints.

PROTOTYPE DEVELOPMENT AND SIMULATION RESULTS

The authors have implemented elements of the solution into a prototype, user-friendly software tool that allows a user to configure the flight/engine operational space and define the independent component parameter variability of interest. The tool then intelligently searches the multi-dimension variability space defined by the flight/engine operational space and component parameter variation bounds to identify worst-case flight/engine operating points in combination with multiple component parameter variations with respect to the selected stability, time domain performance, or frequency domain performance metric.

Test Bed: The initial design and capabilities of the tool were centered around the interaction with a nonlinear engine model developed by the Air Force Research Laboratory (AFRL). This component-based engine model describes all the major engine stages, including fan, high-pressure compressor and turbine, low-pressure turbine, combustor, mixer, and nozzle. All turbo-machinery components, like the fan, compressor, and turbines, are map-driven. A gain scheduled PID controller regulates the amount of fuel flow to the engine based on the difference between N1 demanded and actual speed. In addition to the N1 speed, model inputs include altitude and Mach number. To simulate usage-based degradation of components, the authors modified the model to provide control variability for the following aero-thermal component parameters:

- Combustor Pressure Loss
- Compressor Efficiency Loss
- Turbine Efficiency Loss
- Fuel Flow Loss

Combining these four parameters with the three previously mentioned flight operating condition inputs defines a variability space bounded by a maximum of seven dimensions given the AFRL engine model and its current configuration.

The most prominent candidate for the first step of the optimization procedure is the stability metric. The formulation to find points with the lowest stability margin or the highest value for the stability metric is as follows.

Maximize:

$$J = \|W_2 T\|_\infty \quad \text{(Eq. 10)}$$

Subject to the following constraints:

Plant dynamics:

$$\dot{x} = x_0 + \int_{t_0}^{t_k} f(x(t), u(t))dt \qquad x(t_o) = x_o \quad \text{(Eq. 11)}$$

Plant output:

$$y = g\big(x(t), u(t)\big) \quad \text{(Eq. 12)}$$

Control signal bound:

$$0 \le u^i < \alpha^i \text{ where } i = 1, 2, \cdots, m \quad \text{(Eq. 13)}$$

Given the search space of:

$$h_{low} \le h \le h_{up}$$
$$M_{low} \le M \le M_{up}$$
$$N_{1,low} \le N_1 \le N_{1,up}$$
$$|c_i| \le k_i \qquad k_i \in [\varepsilon, 1] \quad \text{(Eq. 14)}$$

where *x*, *u*, y and are plant states, control inputs, and plant outputs, respectively; α^i is the normalized upper limit of the i^{th} actuator position vector; *h* is altitude with provided upper and lower limits; *M* is Mach number with provided upper and lower limits; *N1* is the low pressure turbine speed with provided upper and lower limits; c_i are the perturbation parameters; and k_i are the perturbation limits. The search space was defined by seven variables as shown in Table 1.

TABLE 1: SEARCH SPACE CONFIGURATION

Search Space Configuration		
Search Variables	Search Space Bounds	
	Lower	**Upper**
N1 Speed	8200 rpm	8800 rpm
Mach	0	0.2
Altitude	0 ft	3000 ft
Combustor Pressure Loss	0 %	4 %
Compressor Efficiency Loss	0 %	4 %
Turbine Efficiency Loss	0 %	4 %
Fuel Flow Loss	0 %	4 %

While the current cost function is the robust stability metric $\|W_2T\|_\infty$, calculated using H_∞ analysis techniques, the tool allows a user to select a cost function or define their own. This feature provides the capability for the user to search the variability space with respect to any control system performance measure.

Optimization Results Using GA

Table 2 shows the optimization results using GA with various population sizes. Figure 3, Figure 4, and Figure 5 depict the performance of GA with populations equal to 15, 50, 100, respectively. While all three simulations reach a similar maximum objective, Simulation 1 calls the least objective function during the optimization, and gives reasonable results. All three simulations indicate strong agreement in identifying the worst case operating conditions.

TABLE 2: OPTIMIZATION RESULTS USING GA WITH VARIOUS POPULATION SIZE

	Simulation 1	Simulation 2	Simulation 3
Population Size	15	50	100
Number of Iterations	375	1150	2000
The Final Maximum Objective	0.2956	0.2989	0.2966
N1 Speed	8200	8200	8201
Mach	0.063	0.063	0.0828
Altitude	0	0	0
Combustor Pressure Loss	2.7%	0%	1.9812%
Compressor Efficiency Loss	2.5826%	1.891%	1.727%
Turbine Efficiency Loss	1.51%	1.1345%	1.201%
Fuel Flow Loss	1.1297%	0%	0.8158%

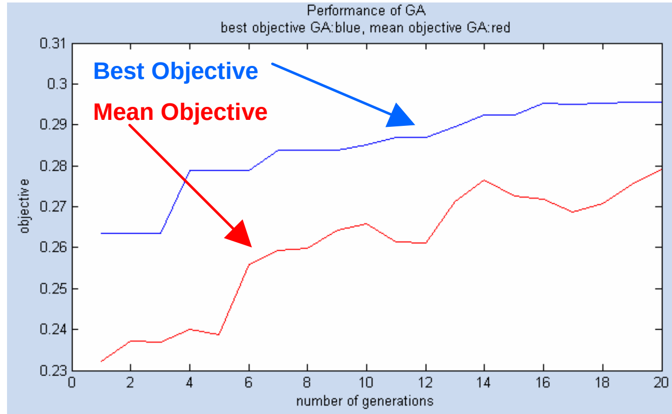

FIGURE 3: GA PERFORMANCE WITH POP. = 15

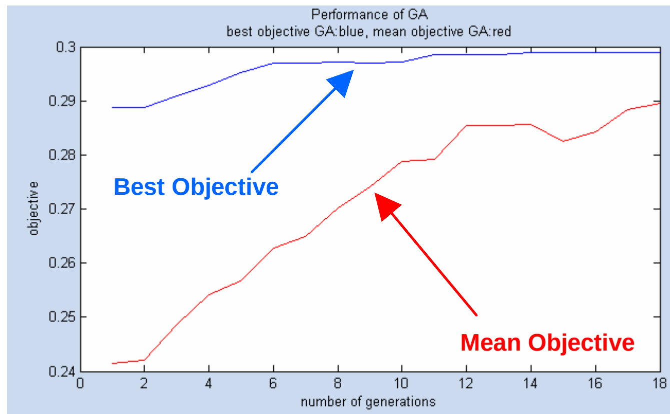

FIGURE 4: GA PERFORMANCE WITH POP. = 50

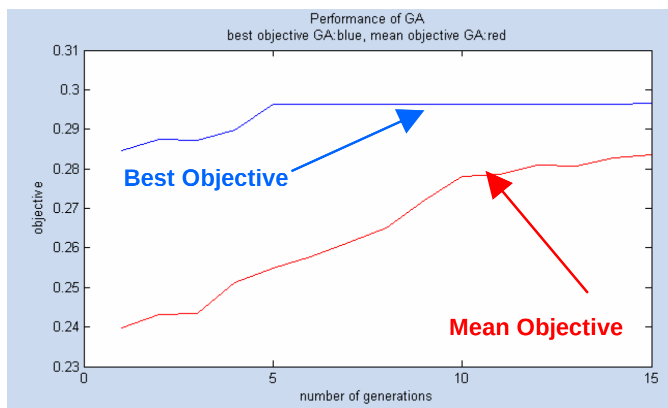

FIGURE 5: GA PERFORMANCE WITH POP. = 100

Optimization Results Using PSO

Table 3 shows the optimization results obtained using PSO for various numbers of particles. Figure 6, Figure 7, and Figure 8 show the trends of the global objective function and particle best throughout the respective simulations. Each PSO simulation produces similar results in terms of objective value and performance in identifying the worst case operating conditions. The identification of worst case parameter variability differs between each of the simulations, as similar to

Table 3: Optimization Results Using PSO With Various Number of Particles

	Simulation 1	Simulation 2	Simulation 3
N Particles	20	50	100
Number of Iterations	400	1000	2000
The Final Maximum Objective	0.289	0.298	0.297
N1 Speed	8209	8200	8228
Mach	0.161	0.063	0.118
Altitude	440	0	0
Combustor Pressure Loss	1.194%	0%	1.239%
Compressor Efficiency Loss	2.463%	1.891%	0%
Turbine Efficiency Loss	1.661%	1.1345%	1.354%
Fuel Flow Loss	1.536%	0%	0.461%

the GA performance. This behavior supports the theory that the change in parameter variability between two points may not form a convex set and can instead be significantly dynamic for nonlinear systems. This was previously identified as a drawback with current techniques. Based on the global trends of the figures, the performance of the PSO algorithm in identifying the worst value objective function depends on the randomness of position for the particle upon each iteration and the particle velocity toward the global best and particle best. The trend in Figure 6 shows several increases in global best during the middle of the simulation, while in Figure 7, the global best is achieved immediately with very minimal increases throughout the remainder of the simulation. In Figure 8, the global best is also achieved early; however, there is an increase at approximately 1600 iterations.

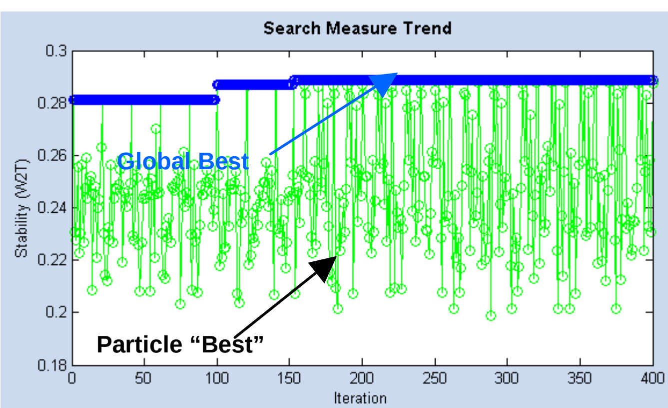

FIGURE 6: PSO PERFORMANCE USING 20 PARTICLES

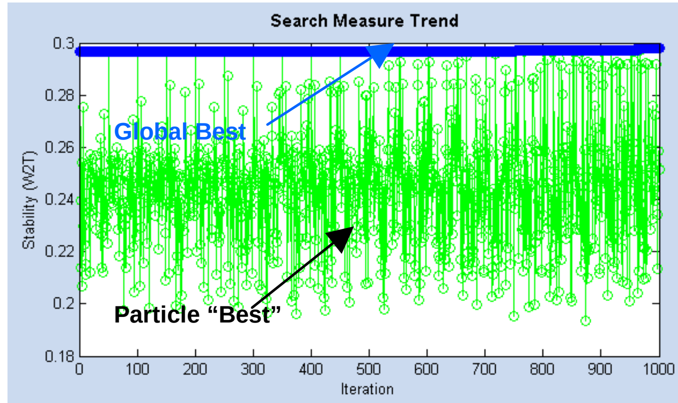

FIGURE 7: PSO PERFORMANCE USING 50 PARTICLES

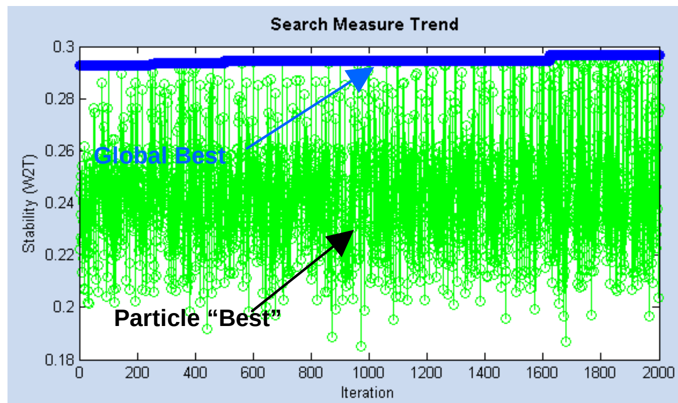

FIGURE 8: PSO PERFORMANCE USING 100 PARTICLES

The performance of the PSO in comparison to GA is complicated not only by the number of particles used for the optimization but also by the configuration of particle velocity toward the global best and particle best. The GA-based optimization provides more consistent results in terms of identifying the worst case operating conditions over the PSO optimization. A extensive Monte Carlo simulation of the hyperspace defined by operating conditions variables in this subregion will identify the true worst case operating conditions and resulting performance and stability metrics. This effort is planned for future work.

CONCLUSION & FUTURE WORK

The authors propose a solution for the analysis of robust stability and performance in identifying variability in propulsion control systems. The solution incorporates a variety of techniques for the analysis and quantification of uncertainty in MIMO, nonlinear control systems. Central to this solution is optimization. Two popular approaches, Genetic Algorithms and Particle Swarm Optimization, were used, to demonstrate and compare the performance of both algorithms for finding the worst variability of operating condition variability and aero-thermal component degradation variability. The authors plan to benchmark the tool against Monte Carlo simulations using a full flight/engine control model to validate the performance of the tool. In addition to investigating the effects of aero-thermal component degradation, the authors plan to assess component degradation of fuel valves or nozzle actuators and their effects on performance and stability robustness. Analyzing these effects on an Integrated Flight and Propulsion Control system model, such as the control system for the JSF STOVL variant, will be of primary interest to enhance the solution for use with advanced aircrafts such as the emerging JSF.

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